THURSDAY, AUGUST 3, 1905.

RECENT FRENCH MATHEMATICAL WORKS.

La Philosophie naturelle intégrale et les Rudiments des Sciences exactes. By Dr. A. Rist. Part i. Pp. vi+132. (Paris: A. Hermann, 1904.) Price 3.50 francs.

Étude sur le Développement des Méthodes géométriques. By Gaston Darboux. Pp. 34. (Paris: Gauthier-Villars, 1904.) Price 1.50 francs.

Sur le Développement de l'Analyse et ses Rapports avec diverses Sciences. By Émile Picard. Pp. 168. (Paris: Gauthier-Villars, 1905.) Price 3.50 francs.

Introduction à la Géométrie générale. By Georges Lechalas. Pp. ix+65. (Paris: Gauthier-Villars, 1904.) Price 1.50 francs.

Introduction à la Théorie des Fonctions d'une Variable. By Jules Tannery. Vol. i. Second edition. Pp. ix+422. (Paris: A. Hermann, 1904.)

Correspondance d'Hermite et de Stieltjes. Edited by B. Baillaud and H. Bourget. Vol. i. Pp. xxi+477. (Paris: Gauthier-Villars, 1905.) Price 16 francs.

THE part which France has played in the development of modern mathematical methods, especially in connection with geometry and analysis, is well known to every mathematician. Of recent years, however, the trend of mathematical thought has considerably changed in every country, and while France has produced a large school of writers on the philosophy of mathematics, it is in the opinion of the present reviewer doubtful whether this school can forge more than a very small link in the chain of mathematical development. The doubts which arose in the minds of mathematicians regarding Euclid's eleventh axiom led to the new science of non-Euclidean geometry, but it was not so much the mere philosophical speculations concerning the axiom itself as the examination of the consequences of making alternative assumptions that led to substantial progress being made. The discovery that we cannot be sure that two and two make four except as the result of experience is undoubtedly of importance, but it is in the development of the consequences of a more extended hypothesis, of which this one is or is not a particular case, that substantial progress must be sought.

Dr. Rist's book may be taken as affording a good example of the kind of philosophical speculations which arise when we try to analyse the why and wherefore of the various processes and operations occurring in even so elementary a subject as arithmetic. It contains chapters on the prolegomena of both geometry and arithmetic, but it is in connection with the latter subject that the discussion is most extended. The mere act of *counting* forms the subject of a number of paragraphs of which the general character may be fairly understood from an enunciation of the headings:—"The number considered as the result of an act," "What do we count?" "Why do we count?" "The different modes of counting." From counting the author proceeds to *calculation,* and in the following chapter gives a detailed discussion of the various processes and symbols involved in the two operations of addition and subtraction. One would naturally expect multiplication and division to be treated in the same way, but instead, Dr. Rist sets out an alternative method of approaching this study, and this first volume closes with a chapter showing how numbers serve for evaluations.

The book seems to appeal more particularly to elementary teachers who only possess a rudimentary training in algebra and geometry, for there is little or nothing in it which assumes more than an elementary knowledge of these subjects. The highly trained mathematician would hardly benefit by reading such a book, as he would probably have already formed ideas of his own on the subject, and in all likelihood would consider the treatment to be unsatisfactory in a good many respects.

Of the useful purpose that can be served by popular addresses containing the survey of wide regions of mathematical thought we have two excellent examples before us. America, with that spirit of internationalism the absence of which from our islands is so greatly to be regretted, loses no chance of picking the brains of the world's greatest mathematicians, irrespective of nationality. Prof. Darboux's pamphlet and the second part of Prof. Picard's contain the substance of addresses delivered at St. Louis last year. The two addresses are to a great extent complementary. Prof. Darboux treats of the development of geometry during the nineteenth century, and Prof. Picard gives a historical account, similar in character, of the development of analysis, with especial reference to its relations with geometry, mechanics, and mathematical physics. Prof. Picard's St. Louis address also forms a sequel to the series of three lectures delivered by him in 1899 at Clark University which form the first part of the same book. The first of these deals with the gradual extension of the meaning attached to the word "function" during the last century, and the numerous new regions of mathematical thought opened up by this development. The second deals with the theory of differential equations, and the third with analytic and certain other functions. In concluding, M. Picard advises students not to specialise in mathematics at too early a stage, but to endeavour to form a general survey of different branches of the science first, and his lectures afford an excellent preliminary step towards the formation of such a survey in the case of analysis.

An English translation of M. Darboux's addresses has appeared in recent numbers of the *Mathematical Gazette.*

M. Lechalas's small volume in the series of "Actualités scientifiques" deals with Euclidean and non-Euclidean geometry. The subject is introduced by a chapter on Euclidean geometry of one, two, and three dimensions. The geometry of Riemann's space is deduced from the Euclidean geometry of four dimensions. That the properties of a Riemann plane and a Euclidean sphere are identical so long as only

the surface itself is concerned is admitted, but whether the Riemann space is identical with, or only analogous to, spherical space in a hyperspace of four dimensions remains a subject of controversy between the author of the book and M. Mansion. At any rate, M. Lechalas does not discuss space of positive curvature independently of its connection with four-dimensional Euclidean space, and accordingly the book contains only one more chapter devoted to the geometry of Lobatchefsky and Bolyaï. In this respect the treatment is analogous to that given in some books on conics where the properties of the ellipse are proved by three-dimensional methods (orthogonal projection) and those of the hyperbola by plane geometry. Whether this is the best plan is open to question; many mathematicians seem to prefer it, and an author cannot please everybody.

In his preface, which is printed in *italics,* M. Tannery fairly well defines the scope and object of his book. Although this is a second edition, it has been entirely re-written. It is primarily intended for readers who do not possess a very extended knowledge of mathematics. It covers mainly those portions of analysis which are commonly found in English text-books on higher algebra, viz. properties of irrational numbers, continued fractions, aggregates, convergency and divergency of series and of infinite products, the binomial theorem, the exponential and logarithmic series, and expansions of trigonometric functions treated without the aid of imaginaries. Finally, we have a chapter on derived functions containing applications of the formula

$$f(x+h)-f(x)=hf'(x+\theta h),$$

and an illustration of functions which have no differential coefficient. The subject-matter may all be included under the heading "functions of real variables treated algebraically," as M. Tannery has avoided the use of geometrical methods in the present volume. A second volume is promised dealing with functions of complex variables, in which geometrical methods are to be freely used.

The treatment is clear and full, and the book gives the impression of being as good an exposition of the subject as could well be written on the lines laid down by the author. It does not profess to give historical or bibliographical information, for which the reader is referred to the "Mathematical Encyclopædia," of which the French edition is now coming out.

An interesting insight into the thoughts of two eminent mathematicians is afforded by the first volume of correspondence between Hermite and Stieltjes, covering the period 1882–1889. The intimacy seems to have arisen in 1882, out of a letter addressed by Stieltjes to Hermite dealing with a theorem of M. Tisserand relating to the expansion of the disturbing force when the mutual inclination of two orbits is considerable. The subject-matter of this letter (which is missing from the collection) was published in the *Comptes rendus* for November 13, 1882.

At this time Thomas Jean Stieltjes was attached to the Observatory of Leyden, and the influence of Hermite doubtless accounts in large measure for his activity in mathematical research during the years which followed, culminating in his migration to France in 1885, after his failure to obtain a mathematical chair in his own country.

A noteworthy feature of Stieltjes's work is his partiality for simple arithmetical tests of general theorems. The value of his examinations of numerical details must have been enormous to a man of Hermite's calibre. It seems as if Hermite in many cases furnished the ideas which Stieltjes elaborated and extended. It was not with Stieltjes alone that Hermite carried on an extensive correspondence, for he was evidently fond of writing letters, and even many of his contributions to journals appeared in epistolary form. But among his various correspondents Stieltjes played a prominent part, and it was Hermite's own wish that the letters of his colleague should be published after the premature death of the latter in 1894. One thing is unfortunately wanting. Hermite was to have written an introduction, but he did not live to do so. In its place we have a preface by M. Picard and a biographical notice by M. H. Bourget, who, in conjunction with M. Baillaud, were colleagues of Stieltjes in the University of Toulouse from 1886 until his death, and who have jointly edited the present volume.

It would be difficult to give a general summary of the subject-matter of this correspondence, which deals with continued fractions, hypergeometric series, Legendre's functions, semi-convergent series, and, indeed, analysis generally. Portraits of Hermite and Stieltjes complete the volume. There is a certain brightness and freshness about the way one of the two mathematicians writes to the other announcing some new result and the second takes up the clue and develops it, and one can imagine the delight that the two kindred spirits must have had in working together.

While the volumes before us are widely different in character, it may be well to warn the busy reader, as has been done on previous occasions, that they all possess one objectionable feature in common. While the guillotine was originally invented in France, the modern instrument of that name has not been applied to its proper use on the pages of any one of the series, consequently readers, unless they are prepared to set up a private guillotine, are compelled to waste hours in hacking and jagging the leaves with a paper knife, producing a very untidy result.

G. H. B.

THE MUTATION THEORY OF THE ORIGIN OF SPECIES.

Species and Varieties: their Origin by Mutation. By Hugo de Vries. Edited by D. T. MacDougal. Pp. xviii+847. (London: Kegan Paul and Co., Ltd., 1905.)

AT the present time, when naturalists are beginning to turn again to the problem of the origin of species, this account of Prof. de Vries's theories and experiments is sure of a welcome, partly